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An attempt to facilitate Observations of Terrestrial Magnetism.

By the Rev. HUMPHREY LLOYD, A. M., M.R.I.A. *Fellow of Trinity College, and Professor of Natural and Experimental Philosophy in the University of Dublin.*

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SINCE the important researches of Baron HUMBOLDT, Captain SABINE, and Professor HANSTEEN, much interest has been awakened on the subject of Terrestrial Magnetism. The intensity of the magnetic force, and the dip, have been determined at various parts of the earth's surface, and are found to follow laws agreeing remarkably with the results of theory. But the experimental data necessary for the complete solution of this important problem are, as yet, far from adequate; and accordingly, the multiplication of observations in remote quarters of the globe, has engaged the zeal of many a scientific individual, and even called forth the resources of more than one state of Europe.

The imperfections of our instruments of observation have long presented a formidable obstacle to the advancement of these inquiries. Many of these have indeed been guarded against by the skill of the observer, and by the application of suitable corrections. There are still, however, some inherent sources of fallacy in the instruments themselves, which seem almost inseparable from them in their ordinary form. The defects of the common dipping needle have been long acknowledged, and still continue to embarrass observers. The chief of these defects arises from the nature of the suspension. The directive force, it is well known, diminishes with the angular distance from the true line of the dip, varying as the sine of that distance. Hence when the needle is near the position which it would assume under the influence of the terrestrial action, if free to move, the directive tendency may be so small as to be more than balanced by the friction of the axle against its supports; and in this case the needle will rest out of the position in which the earth's force would place it. This evil will of course be increased by any irregularity in the form of the axle itself, and such irregularities can never be wholly removed. To this is added another source of error, arising from the non-coincidence of the axle with the centre of gravity of the needle. Where this fault exists, gravity unites with magnetism in determining the position of equilibrium of the needle, and it will assume a direction different from that of the resultant of the terrestrial magnetic forces. The error arising from this cause, it is true, is in a great degree corrected by the process of reversing the poles of the needle, in the usual mode of observation; but still it cannot be said to be wholly removed, nor are the methods of correction altogether unobjectionable.

These and other difficulties in the direct determination of the dip, have led to the occasional employment of *indirect* methods. Thus, if the total magnetic intensity be known, together with its vertical or its horizontal component, the dip is readily found by elimination. A method founded on this principle was suggested by LAPLACE, and practised by HUMBOLDT and Captain SABINE. The total intensity was determined by noting the time of vibration of a dipping needle, oscillating in the plane of the magnetic meridian; and the vertical component was similarly obtained from the time of vibration of the same needle in the plane at right angles to the former: the ratio of these two forces is equal to the sine of the dip. Captain SABINE has also employed another method derived from the same principle. The horizontal component of the terrestrial magnetic force was found by observing the time of vibration of the needle suspended horizontally: and this component, divided by the total intensity, is equal to the cosine of the dip. The dipping needle, however, seems to be badly adapted to observations of vibration. Owing to the friction of the axis, the needle is soon brought to rest, unless the initial arc of vibration be considerable; and, when this is the case, the observer is involved in many sources of error, against which it seems difficult to guard.

A method similar to these in principle, but free from the objections just stated, was adopted many years since by COULOMB. In this method, the elements obtained by observation were the vertical and horizontal components of the intensity, the ratio of which is equal to the tangent of the dip. The horizontal component was determined, as in the last case, by observing the number of oscillations performed in a given time by the needle suspended horizontally; while the vertical component was found by determining the weight of a counterpoise, placed at a given distance on the southern arm of the needle, and sufficient to bring it to the horizontal position. The moment of this counterpoise, or its weight multiplied by its distance from the axle, is obviously equal to the vertical component of the magnetic intensity. M. BIOT considers this as the most accurate method that has been ever applied to the determination of the dip. It seems, however, open to very serious objections, which have been pointed out by M. POUILLET. In order to determine the horizontal component from the time of vibration, so that it may be comparable with the other component determined *statically*, it is necessary that the moment of inertia of the needle should be known a priori; and that this should be, the needle must have a regular mathematical form.

That used by Coulomb was of uniform and very small dimensions throughout its entire length, and in the calculation was treated as a mathematical line. There are other difficulties connected with the practice of this method, on which it is unnecessary to dwell: so that, notwithstanding its high sanction, it has not, as far as I am aware, been adopted by any other observer.

I have alluded to the method of Coulomb in this place, because it suggested that which I am now about to propose. It occurred to me that the components of the magnetic intensity might *both* be determined *statically*, and by one and the same process. Thus all the objections, to which Coulomb's method is liable, would be avoided, and the process itself rendered much simpler of application, as well as more accurate in its results.

Let us conceive a magnetic needle suspended as the ordinary dipping needle, and placed in the magnetic meridian; and let us calculate the directive effect of the terrestrial magnetic force upon it in any position.

If q denote the quantity of the magnetic fluid in any point of the needle, and ϕ the terrestrial magnetic force acting on the unit of quantity, $\phi\, q$ will be the force exerted on the point in question. Let δ denote the angle which the direction of this force makes with the horizon, or the *dip*, and let θ be the angle formed by the needle itself with the horizon: then $\delta-\theta$ is the angle which the direction of the force makes with the needle; and consequently, the moment of the force tending to turn the needle round its axle is $\phi\, q\, r \sin(\delta-\theta)$; r being the distance of the point in question from the axle. Now, to obtain the total effect of these forces acting on all the points of the needle, we have only to multiply this moment by the element of the mass, dm, and integrate the result. The total directive effect of the earth upon the needle is therefore

$$\int \phi\, qrdm. \sin(\delta-\theta) = \phi \sin(\delta-\theta) \int qrdm.$$

The integral $\int qrdm$, in this expression, will depend on the form of the needle, and on the law of the magnetic distribution in it. Let its value, taken within the limits of the dimensions of the needle, be denoted by σ: then the moment of the terrestrial magnetic force, tending to turn the needle round its axle, is expressed by

$$\phi\, \sigma \sin(\delta-\theta).$$

Now, if a weight be placed on the southern arm of the needle, its effect to turn the needle in the opposite way is

$$\mu \cos\theta;$$

μ being the *moment* of the mass, or the weight multiplied by its distance from the axle. Accordingly, in the case of equilibrium, we have $\mu \cos\theta = \phi\sigma \sin(\delta-\theta)$

Or, $$\mu = \phi\, \sigma\, (\sin\delta - \cos\delta \tan\theta) \qquad (1)$$

Now, let us suppose two weights to be placed in succession on the southern arm of the needle, and let their moments, μ' and μ'', be adjusted so as to bring the needle into the positions forming half a right angle with the horizon, below and above; the corresponding values of θ then are $+45^\circ$ and -45°, and substituting, we have the two equations following

$$\mu' = \phi\,\sigma\,(\sin\delta - \cos\delta), \qquad \mu'' = \phi\,\sigma\,(\sin\delta + \cos\delta).$$

Adding and substracting these equations, we have

$$\mu'' + \mu' = 2\,\phi\,\sigma\,\sin\delta, \qquad \mu'' - \mu' = 2\,\phi\,\sigma\,\cos\delta.$$

from which we obtain,

$$\tan\delta = \frac{\mu'' + \mu'}{\mu'' - \mu'}, \qquad \phi\,\sigma\,\sqrt{2} = \sqrt{\mu''^2 + \mu'^2} \qquad (2).$$

Accordingly, to determine the intensity of the terrestrial magnetic force, as well as the dip, we have only to find the moments of two counterpoises which will bring the needle into the two positions above mentioned; the sum of these divided by their difference is equal to the tangent of the dip, and the square root of the sum of their squares furnishes a measure of the intensity. Another set of results might be obtained by observing also the moment of the counterpoise which brings the needle to the horizontal position; for it will be easily seen that this moment is equal to the arithmetical mean between the two preceding, or equal to $\phi\,\sigma\,\sin\delta$.

In the practice of this method, it is obvious that a divided circle is not required; it is only necessary to place four lines on the frame of the instrument corresponding to the inclinations of 45°. It is requisite, however, to have some means of determining the moments of the counterpoises with facility. For this purpose I procured a needle furnished with a small sliding weight, and having a scale on its southern arm, divided to the 1-50th of an inch. I had then only to observe the division on the scale coinciding with a fixed mark on the sliding weight, when the needle was brought to the two required positions.

Simple as this method appears, a few trials were sufficient to convince me that it would prove a most inconvenient one in practice. The adjustment of the counterpoise, so as to bring the needle to the required inclinations, was found to be a tedious and delicate operation, requiring a frequent removal of the needle from its supports; and the repetition of this process, in the various positions of the needle and instrument in which an observation of dip is usually taken, multiplies, of course, the difficulty. For these, and other reasons, it appeared far preferable to observe the *positions* which the needle assumes with *fixed counterpoises*, rather than the counterpoises which bring it into fixed positions. It will be easily seen, that two such observations are sufficient to determine the dip and the magnetic intensity.

Let μ' and μ'' be the *moments* of the two fixed counterpoises, and θ' and θ'' the corresponding inclinations of the needle to the horizon: then, substituting in equation (1),

$$\mu' = \phi\,\sigma\,(\sin\delta - \cos\delta\tan\theta'), \qquad \mu'' = \phi\,\sigma\,(\sin\delta - \cos\delta\tan\theta'').$$

We have thus two equations containing ϕ and δ, from which these quantities may be obtained by elimination. To effect this, let the former be divided by the latter, and we find

$$\frac{\mu'}{\mu''} = \frac{\tan\delta - \tan\theta'}{\tan\delta - \tan\theta''}, \qquad \tan\delta = \frac{\mu'\tan\theta'' - \mu''\tan\theta'}{\mu' - \mu''};$$

And denoting the constant factors $\frac{\mu'}{\mu'-\mu''}$ and $\frac{\mu''}{\mu'-\mu''}$ by ν' and ν'',

we have finally,

$$\tan\delta = \nu'\tan\theta'' - \nu''\tan\theta'. \qquad (3)$$

If we eliminate δ between this equation and either of the two equations given above, we readily obtain an expression for ϕ in terms of μ', μ'', and θ', θ''; but as the resulting expression is somewhat complicated, it will be much simpler in practice to obtain the dip, in the first instance, from the equation last deduced, and to substitute its value in the formula

$$\phi = \frac{\mu\cos\theta}{\sigma\sin(\delta - \theta)}.$$

The quantity σ, in this formula, depending on the law of distribution of the magnetic fluid in the needle employed, is unknown; and, consequently, the *absolute intensity* of the terrestrial magnetic force cannot be determined without some other artifices. In general, however, the ratio of this force at different parts of the earth's surface is alone required; and for this purpose the force at some given place (usually the magnetic equator) is taken as unity, and the force at other places determined by comparison. Let $\phi_{,}$ be the force at the given place, and $\delta_{,}$ $\theta_{,}$ the corresponding values of δ θ, then

$$\phi_{,} = \frac{\mu\cos\theta_{,}}{\sigma\sin(\delta_{,} - \theta_{,})},$$

and dividing

$$\frac{\phi}{\phi_{,}} = \frac{\sin(\delta_{,} - \theta_{,})\cos\theta}{\sin(\delta - \theta)\cos\theta_{,}}. \qquad (4)$$

The counterpoises I employed, in the practice of this method, were small pieces of brass wire inserted in holes which were drilled at different distances along the axis of the needle. Finding that the weights which were sufficient to balance the magnetic force, at a moderate distance from the axle, were unmanageably small, I used two counterpoises, one of which was a weight of one grain, placed on the southern arm, at the distance 1.38 inches from the axle; the other was a weight of .7 of a grain, placed on the northern arm, at the distances 1.1 and 1.38 inches successively. Thus the moments of the actual counterpoises were the differences of those on the two arms, added to the difference of the moments of the arms themselves.*

It was now necessary to determine with accuracy the ratio of these moments; and, as this could hardly be effected with the requisite precision by any direct measurements, I determined to use for the purpose the means suggested by the proposed method of observation itself. This method, it has been shown, determines the dip by means of the two positions of the counterpoised needle, when the ratio of the moments is known: therefore, conversely, if the dip be otherwise found, the same formulæ may be used to calculate the ratio of the moments. The following observation was accordingly taken for this purpose:

Killiney, Sept. 11. *Needle I.*†

Marked end	Limb	E marked side		W	
		N	S	N	S
N	E	71° 7′	71° 15′	71° 15′	71° 30′
	W	70 45	70 45	70 50	70 50
S	E	71 10	71 20	71 30	71 45
	W	70 45	70 40	71 10	71 5
Mean		70° 56′.8	71° 0′	71° 11′.3	71° 17′.5
		Final Mean = 71° 6′.4.			

Killiney, Sept. 12. *Needle II.*†

Counterpoise	Limb	E marked side		W	
		N	S	N	S
I	E	−18° 45′	−18° 37′	−19° 0′	−18° 52′
	W	−20 25	−20 15	−21 0	−21 0
II	E	+23 10	+23 20	+25 40	+25 55
	W	+22 30	+22 35	+25 20	+25 25
Mean		$\theta' = -19° 44'$		$\theta'' = +24° 14'$	

Substituting the values of δ, θ' and θ'', thus obtained, in the equation,

$$\frac{\mu'}{\mu''}=\frac{\tan\delta-\tan\theta'}{\tan\delta-\tan\theta''};$$

we find the ratio of the connterpoises to be 1.3274.

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* The moments of the two arms of the needle were rendered unequal by the weights which were unsymmetrically abstracted in the process of drilling the holes.

† Needle I. is the common needle—Needle II. the counterpoised needle.

It now remained to take a series of observations, with the common as well as the counterpoised needle, and to compare their results. The following tables contain the results of two such series. The first series of observations was taken at places differing in dip: the second series at the same place. The poles of the counterpoised needle were not reversed, as such a process would be incompatible with one of the objects of the needle—namely, the determination of the variations of terrestrial intensity; and it fortunately happens, that this troublesome operation is, in the present method, wholly unnecessary. The use of reversing the poles in the common needle, it is well known, is to correct for the moment of the needle itself. If the centre of gravity of the needle deviates at all from the centre of motion, *in the direction of its length*, the moment of the needle will increase or diminish its inclination to the horizon, according as the deviation is towards the north or south pole; and this error can only be corrected by reversing the poles of the needle, so that its moment may act in opposite directions in the two cases.* But, in the needle proposed in these pages, the centre of gravity of the whole is made intentionally to deviate from the axle, and the dip determined from the altered inclinations. In this case, accordingly, the moment of the needle, if any, simply increases or diminishes the moments of the actual counterpoises; and, as these are determined *a posteriori*, the moment of the needle itself will of course be included in the determination.

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* It may be readily shown, that the tangent of the true dip, is the arithmetical mean between the tangents of the observed inclinations of the needle, when the same end is a north and south pole. But when the difference of the observed inclinations is small, we may, without much error, suppose the same relation to hold amongst the angles themselves.

TABLE I.—*Containing Observations of the positions of the Common and Counterpoised Needle, at places differing from one another in Dip.*

		Observatory, Armagh, Sept. 16.				*Dundonnell, County of Down, Sept.* 23.			
	Marked end Limb	E	marked side	W		E	marked side	W	
		N	S	N	S	N	S	N	S
NEEDLE I	N E	72° 5′	72° 5′	71° 55′	71° 55′	72° 0′	72° 5′	71° 55′	72° 0′
	N W	71 22	71 10	71 25	71 15	71 15	71 7	71 30	71 20
	S E	72 0	72 4	72 5	72 15	71 55	72 0	72 0	72 5
	S W	71 25	71 10	71 20	71 5	71 20	71 15	71 22	71 15
	Mean	71 43	71 37.2	71 41.2	71 37.5	71 37.5	71 36.8	71 41.8	71 40
		Final mean = 71° 39′.7				Final mean = 71° 39′			
	Counter-poise Limb	E	marked side	W		E	marked side	W	
		N	S	N	S	N	S	N	S
NEEDLE II	I E	−21° 20′	−21° 15′	−22° 10′	−22° 7′	−21° 10′	−21° 5′	−22° 50′	−22° 50′
	I W	−21 25	−21 15	−22 5	−21 55	−22 0	−21 55	−22 52	−22 45
	II E	+22 30	+22 30	+25 40	+25 35	+22 5	+22 5	+25 15	+25 20
	II W	+22 0	+22 5	+23 45	+23 45	+22 7	+22 7	+24 40	+24 40
	Mean	θ′ = − 21° 41′.5		θ″ = + 23° 28′.8		θ′ = − 22° 11′		θ″ = +23° 32′	
		Calculated dip = 71° 26′				Calculated dip = 71° 38′			

		Museum, Belfast, Sept. 28.				*Garden, Trinity College, Oct.* 21.			
	Marked end Limb	E	marked side	W		E	marked side	W	
		N	S	N	S	N	S	N	S
NEEDLE I	N E	72 20	72 22	72 15	72 15	71 16	71 16	71 10	71 21
	N W	71 30	71 25	71 30	71 28	70 43	70 35	70 33	70 30
	S E	72 15	72 20	72 10	72 15	71 15	71 27	71 18	71 22
	S W	71 35	71 37	71 30	71 20	70 36	70 34	70 35	70 32
	Mean	71 55	71 56	71 51.2	71 49.5	70 57.5	70 58	70 54	70 56.2
		Final mean = 71° 52′.9				Final mean = 70° 56′.4			
	Counter-poise Limb	E	marked side	W		E	marked side	W	
		N	S	N	S	N	S	N	S
NEEDLE II	I E	−21° 10′	−21° 0′	−21° 45′	−21° 35′	−23° 27′	−23° 30′	−23° 42′	−23° 41′
	I W	−22 7	−22 7	−22 37	−22 30	−24 15	−24 17	−25 7	−25 7
	II E	+22 40	+22 40	+25 30	+25 35	+20 27	+20 27	+21 40	+21 43
	II W	+22 52	+22 45	+25 30	+25 40	+19 52	+19 52	+22 37	+22 32
	Mean	θ′ = − 21° 51′		θ″ = +24° 9′		θ′ = −24° 8′.2		θ″ = +21° 8′.8	
		Calculated dip = 71° 49′				Calculated dip = 71° 12′			

TABLE II.—*Containing Observations of the positions of the Common and Counterpoised Needle, taken in the Philosophy School of Trinity College.*

NEEDLE I

		October 22. Therm.=56½°				*October* 23. Therm.=57°			
Marked end Limb		E marked side		W		E marked side		W	
		N	S	N	S	N	S	N	S
N	E	71° 20′	71° 20′	71° 15′	71° 20′	71° 15′	71° 7′	71° 15′	71° 22′
	W	70 40	70 30	70 25	70 30	70 35	70 30	70 35	70 30
S	E	71 15	71 22	71 15	71 22	71 15	71 20	71 15	71 25
	W	70 35	70 30	70 50	70 50	70 35	70 25	70 30	70 15
Mean		70 57.5	70 55.5	70 56.2	71 0.5	70 55	70 50.5	70 53.8	70 53
		Final mean=70° 57′.4				Final mean=70° 53′.1			

NEEDLE II

		October 22.				*October* 23.			
Counter-poise Limb		E marked side		W		E marked side		W	
		N	S	N	S	N	S	N	S
I	E	−27° 52′	−27° 52′	−29° 20′	−29° 22′	−12° 30′	−12° 22′	−14° 15′	−14° 30′
	W	−28 45	−28 45	−28 52	−28 52	−13 37	−13 37	−15 55	−15 50
II	E	+16 15	+16 15	+20 15	+20 25	+26 7	+26 15	+29 35	+29 45
	W	+15 15	+15 15	+19 0	+19 0	+26 0	+26 0	+30 10	+30 10
Mean		$\theta' = -28^\circ\,42'.5$		$\theta'' = +17^\circ\,42'.5$		$\theta' = -14^\circ\,4'.5$		$\theta'' = +28^\circ\,0'.2$	
		Calculated dip = 71° 22′.5				Calculated dip = 71° 6′			

NEEDLE I

		(1) *October* 24. Therm.=58½°				(2) *October* 24. Therm.=59°			
Marked end Limb		E marked side		W		E marked side		W	
		N	S	N	S	N	S	N	S
N	E	71° 15′	71° 22′	71° 7′	71° 22′	71° 0′	71° 15′	71° 10′	71° 25′
	W	70 40	70 30	70 30	70 22	70 45	70 45	70 30	70 30
S	E	71 15	71 20	71 7	71 15	71 20	71 40	71 7	71 20
	W	70 35	70 25	70 30	70 30	70 35	70 30	70 35	70 30
Mean		70 56.2	70 54.2	70 48.5	70 52.2	70 55	71 2.5	70 50.5	70 56.2
		Final mean=70° 52′.8				Final mean=70° 56′			

NEEDLE II

		(1) *October* 24.				(2) *October* 24.			
Counter-poise Limb		E marked side		W		E marked side		W	
		N	S	N	S	N	S	N	S
I	E	−35° 30′	−35° 45′	−38° 10′	−38° 15′	−10° 45′	−10° 40′	−12° 45′	−12° 45′
	W	−38 7	−38 0	−39 55	−39 55	−12 45	−12 45	−14 0	−14 0
II	E	+ 6 35	+ 6 37	+ 8 45	+ 8 50	+26 30	+26 30	+30 30	+30 30
	W	+ 4 15	+ 4 20	+ 6 30	+ 6 30	+27 0	+27 0	+30 30	+30 30
Mean		$\theta' = -37^\circ\,57'$		$\theta'' = +6^\circ\,32'.8$		$\theta' = -12^\circ\,33'$		$\theta'' = +28^\circ\,37'.5$	
		Calculated dip = 70° 39′				Calculated dip = 70° 56′			

From the first of these Tables we see that the values of the dip, obtained by the common and counterpoised needle, do not differ from one another by 16′, at any of the four stations; while at two of the stations the difference is considerably within the limits of error of a single observation of dip, taken in the usual manner.

The general agreement of the results of the two methods being thus established, the second series of observations, given in Table II, enables us to compare the methods with respect to accuracy; for these observations being all taken at the same place, the consistency of the results will afford the means of estimating the probable limit of error in each method.

Comparing then, the results of Table II, it will be seen in the first place, that the mean of the four observations made with Needle I, gives 70° 55′.1 for the dip, in the Philosophy school of Trinity College; while the mean of the observations taken with Needle II, is 71° 0′.9, differing from the former by less than 6′.

The results of Needle II, are, however, by no means as consistent among themselves as those of Needle I. The greatest difference between any one result and the mean, with the common needle, is only 2½′; while, with the counterpoised needle, the corresponding difference amounts to 22.′ This, however, can scarcely be regarded as decisive against the accuracy of the method. The common needle was one of remarkable nicety of construction; while that used with the counterpoises, was obviously erroneous. It was necessary for the consistency of the results that the points of application of the counterpoises, should coincide exactly with the magnetic axis, or central line of the needle. Now, no very precise means were adopted to insure this coincidence, in drilling the holes for the counterpoises; and that the coincidence was in fact not effected, will readily appear on looking over the results. It will be seen that, when the second counterpoise is employed, there is a difference amounting sometimes to 4° between the angles read off with the face of the needle in opposite positions; and this plainly indicates an error in the place of the second hole on the northern arm of the needle. It is to be observed, further, that the magnetic state of the needle was widely different in the several observations, having been altered before each trial by a pair of bar magnets; so that the method was subjected to a more severe test than any to which it could be exposed in practice. Even with these disadvantages, however, the values of the dip obtained with this needle do not appear to differ from one another more than is common in such observations; so that the result of the trial cannot be regarded as unfavorable.

During the progress of the observations recorded in Table II, I made a cotemporaneous series of observations on the rate of vibration of Needle II, suspended horizontally; with the view of ascertaining how far the results obtained with this needle could be relied on for the determination of the force. For this purpose the magnetism of the needle was altered by a pair of bar magnets, and its rate of vibration ascertained in the usual manner, after each change in its magnetic condition, by a good chronometer. The observation of the angles of position of the same needle, when counterpoised, was taken immediately before or after, and, as nearly as possible, at the same temperature. But unfortunately the moment of inertia of the needle was unguardedly altered during the observations, so that no reliance could be placed on the results.

Since the preceding trials were made, it appeared to me desirable to examine theoretically the accuracy of the counterpoised needle in its different positions, and to ascertain, if possible, the most eligible. It will appear from the foregoing observations, that between two distant places, the *range*, or change of position of the counterpoised needle, is considerably greater than the corresponding change in the dip. This I had been previously led to expect from theory, and had found that the change of position, corresponding to a given change in the dip, would be greatest when $\tan 2\,\theta = \frac{1}{4} \tan \delta$. When $\delta = 70°$, this formula gives, for the *angle of maximum range*, $\theta = 17° \, 15'$; and the ratio of the range itself to the corresponding variation in the dip was found, in this case, to amount to 3.275. I was thus led to expect that changes of dip, which would be inappreciable by the common needle, might be detected with this. It appeared, however, on trial, that the *limits of error* were likewise increased in this needle; so that it was necessary to examine it in another point of view.

Resuming the original equation

$$\mu \cos \theta = \phi \, \sigma \, (\delta - \theta),$$

and differentiating and dividing by the equation itself, considering θ, δ and ϕ as all variable, we find

$$\cos \theta \, \sin (\delta - \theta) \, \frac{d\phi}{\phi} + \cos \theta \, \cos (\delta - \theta) \, d\delta - \cos \delta \, d\theta = 0.$$

This equation will give the error in the dip, or in the force, corresponding to a given error in the position of the counterpoised needle ; for, making $d\phi$ and $d\delta$, successively, equal to nothing, there is

$$d\delta = \frac{\cos \delta \, d\theta}{\cos \theta \cos (\delta - \theta)}, \qquad \frac{d\phi}{\phi} = \frac{\cos \delta \, d\theta}{\cos \theta \sin (\delta - \theta)}. \tag{5}$$

Hence, supposing the error in the position of the needle, $d\theta$, to be given, we can find the directions of the needle, in which the *resulting error* in the determination of the dip, or of the force, shall be a *minimum*. It is easily seen that $d\delta$ is a minimum, when $\theta = \frac{1}{2}\delta$; and that $\frac{d\phi}{\phi}$ is least, when $\theta = \frac{1}{2}\delta - 45°$. Hence, in our latitudes, for the determination of the dip and of the force, the counterpoises should be such as to bring the needle into the positions forming the angles $+35°$ and $-10°$, respectively, with the horizon.

Such then should be the inclinations of the needle, in order that any *constant error of position*, $d\theta$, may have the smallest influence on the calculated dip and intensity. Of such constant errors the most obvious is the error to which the observer is subject in reading the angle on the limb, arising from the smallness or imperfection of the divisions. But the *error of reading* is not the only, or even the most important error, to which we are liable in determining the position of the needle. It has been already stated that, owing to the friction of the axle, the needle is often brought to rest out of its true direction: now the error of position arising from this cause is, in instruments of the usual size, of greater magnitude than the error of reading, and that magnitude is different in the different positions of the needle.

In order to determine the amount of this error, it will be necessary to consider the directive force, by which the needle is urged to its position of equilibrium. This force is obviously the difference between the magnetic moment, $\phi\,\sigma \sin(\delta-\theta)$, and the moment of the counterpoise, $\mu \cos\theta$; so that, if its magnitude be denoted by F,

$$F = \phi\,\sigma \sin(\delta-\theta) - \mu\cos\theta.$$

But, if $\theta_{\prime}$ be the position of equilibrium, there is $\phi\,\sigma\sin(\delta-\theta_{\prime})-\mu\cos\theta_{\prime}=0$; and substituting the value of μ, obtained from this equation, in the preceding formula, and observing that

$$\sin(\delta-\theta)\cos\theta_{\prime}-\sin(\delta-\theta_{\prime})\cos\theta=\cos\delta\,(\sin\theta_{\prime}\cos\theta-\cos\theta_{\prime}\sin\theta)=\cos\delta\sin(\theta_{\prime}-\theta),$$

we have finally

$$F=\phi\,\sigma\frac{\cos\delta}{\cos\theta_{\prime}}\sin(\theta_{\prime}-\theta).$$

Hence the directive force varies as the sine of the angular distance from the position of equilibrium. Accordingly, when that angular distance is reduced to a certain limit, the force becomes equal to the friction, and is balanced by it. Let ε be the magnitude of the angle, $\theta_{\prime}-\theta$, when the directive force becomes equal to the friction, f; then $f=\phi\,\sigma\frac{\cos\delta}{\cos\theta}\sin\varepsilon=\phi\,\sigma\frac{\cos\delta}{\cos\theta}\varepsilon$, since ε is small; and therefore

$$\varepsilon=\frac{f\cos\theta}{\phi\,\sigma\cos\delta}. \qquad (6)$$

The angle ε, thus found, is obviously the *limit of error* to which we are liable in determining the position of the needle, arising from friction. Its value depends, as we see, upon the force of friction, the intensity of the terrestrial magnetic force, the magnetic moment of the needle, and its position with relation to the dip. In order to determine the *resulting error* which it will produce, in the determination of the dip and intensity, we have only to substitute its value for $d\theta$, in the equations (5). We find, in this manner,

$$d\delta = \frac{f}{\phi\,\sigma\cos(\delta-\theta)}\,, \qquad \frac{d\phi}{\phi} = \frac{f}{\phi\,\sigma\sin(\delta-\theta)}\,;$$

or, denoting the limit of error arising from friction in the ordinary position of the needle by ϵ',

$$d\delta = \epsilon'\sec(\delta-\theta)\,, \qquad \frac{d\phi}{\phi} = \epsilon'\operatorname{cosec}(\delta-\theta)\,. \qquad (7)$$

We learn, then, that the error in the determination of the dip, arising from friction, is least when $\delta-\theta=0$; and that the smallest value of $d\delta$ is ϵ'.* The corresponding error in the determination of the force will be a minimum, when $\delta-\theta=90°$; in which case $\frac{d\phi}{\phi} = \epsilon'$, ϵ' being expressed in parts of radius.

As far then, as friction is concerned, it would appear to be the most advantageous modification of the method suggested in the preceding pages, to observe the position of the needle, in the first instance, without any counterpoise, and, afterwards, with a counterpoise which will bring it into a position nearly perpendicular to the line of the dip. The former of these angles is the dip itself; and the two angles, when substituted in formula (4), furnish the measure of the intensity. But, in order to avoid the error arising from the non-coincidence of the centre of gravity with the axle, I think it would be far better to use a small counterpoise in the first instance; or even to consider the moment of the needle itself, (or its weight multiplied by the distance of its centre of gravity from the axle,) as a counterpoise acting with or against the magnetic moment. The ratio of this moment to that of the other counterpoise should, of course, be determined by the indirect method which has been already explained.†

We may now form an estimate of the accuracy of this method, as compared with the usual one, in the determination of the magnetic intensity. In the received method, it is well known, the horizontal component of the force is determined by observing the time of vibration of the needle suspended horizontally. Now let us suppose this portion of the force to be *completely* determined, and inquire how far the probable error in the dip will affect the total intensity, thence deduced. If h denote the horizontal component, we have

$$h = \phi\,\sigma\cos\delta\,;$$

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* We have here considered the effect of friction on the result, so far as it depends upon a single reading. The eight readings usually taken may undoubtedly diminish still further the resulting error of dip; but as these readings are taken in order to correct other errors, we have disregarded their effect here. If, however, the needle be so perfect in its construction, that the errors arising from the non-coincidence of the centre of gravity with the axle, and the deviation of the magnetic axis from the axis of the needle, &c. are less than the error of friction, then the multiplication of readings will have the effect of reducing the latter in the ultimate result.

† See page 6.

and differencing and dividing, h being considered as constant,

$$\frac{d\phi}{\phi} = d\delta \tan \delta.$$

Now, confining our consideration to the error arising from friction, $d\delta = \frac{f}{\phi\sigma} = \epsilon'$; so that the *limit of error* in the determination of the force, in the usual method, is $\epsilon' \tan \delta$; while, in that now proposed, it may be reduced to ϵ'. The limit of error in the common method, therefore, is to that of the method now proposed, in the ratio of the tangent of the dip to unity; that is, in our latitude, as 2.75 to 1, nearly.

In the instrument in my possession, constructed by Mr. Robinson, of Devonshire-street, London, I consider the limit of error arising from friction, in the ordinary position of the needle, not to exceed 5′. With this instrument, therefore, $\epsilon' = 5' = .0015$, nearly, radius being unity; so that the error in the determination of the force does not exceed the .0015 part of the entire quantity.

In order to satisfy myself more fully as to the accuracy of this method, the following series of observations was taken. The position of the needle (Needle I of the preceding experiments) was observed, first without any counterpoise, and secondly with a counterpoise which brought it into a position nearly perpendicular to the line of the dip. The observations were taken in the same spot of the room, and as nearly as possible at the same temperature; the magnetism of the needle was not interfered with during the observations.

TABLE III. *Containing the results of Four Observations taken with Needle I. in the Philosophy School of Trinity College. Therm.* = 61°.5.

Angle Limb		I.	marked side			II.	marked side		
		E		W		E		W	
		N	S	N	S	N	S	N	S
δ	E	71° 7′	71° 18′	71° 12′	71° 25′	71° 5′	71° 10′	71° 10′	71° 25′
	W	70 35	70 30	70 25	70 22	70 30	70 22	70 30	70 28
θ	E	−17 30	−17 30	−15 15	−15 7	−17 0	−17 7	−15 15	−15 7
	W	−19 40	−19 37	−18 15	−18 5	−19 40	−19 45	−18 15	−18 7
Mean		δ = 70° 51′.8,		θ = − 17° 37′.4		δ = 70° 50′,		θ = −17° 32′	

Angle Limb		III.	marked side			IV.	marked side		
		E		W		E		W	
		N	S	N	S	N	S	N	S
δ	E	71° 0′	71° 10′	71° 15′	71° 30	71° 5′	71° 15′	71° 10′	71° 20′
	W	70 30	70 25	70 30	70 25	70 30	70 22	70 30	70 30
θ	E	−16 55	−17 0	−15 15	−15 7	−17 22	−17 35	−15 20	−15 20
	W	−19 30	−19 30	−17 45	−17 45	−19 45	−19 45	−17 55	−17 55
Mean		δ = 70° 50′.6,		θ = − 17° 20′.9		δ = 70° 50′.2,		θ = − 17° 37′.1	

The mean values of δ and θ, as deduced from these four observations, are

$$\delta_{,} = 70^\circ.\ 50'.7\ ,\quad \theta_{,} = -17^\circ.\ 31'.8\ ;$$

and if we take the force corresponding to these mean values as unity, the numerical values of the force, derived from each separate observation, will be obtained by substituting the corresponding values of δ and θ, given by the preceding table, in the formula

$$\log \phi = \log \cos \theta - \log \cos \theta_{,} + \log \sin (\delta_{,} - \theta_{,}) - \log \sin (\delta - \theta).$$

We find in this manner

$$\phi_1 = .9994\ ,\quad \phi_2 = 1.0000\ ,\quad \phi_3 = 1.0011\ ,\quad \phi_4 = .9995.$$

The greatest deviation of these values from the mean is .0011; so that the limit of error in the determination of the force, would appear from these observations to be little more than the one-thousandth part of the entire quantity.

It appears then from the preceding, that the Terrestrial Magnetic Intensity may be determined, together with the Dip, with the aid of a single instrument, and by a process even somewhat less troublesome than that by which the dip alone is usually determined.* To any one who considers the numerous precautions required, in the common method of determining the magnetic intensity, the saving of time and labour thus effected will be abundantly obvious. But it is an advantage of much greater moment, that the results of the proposed method, so far as the intensity is concerned, will be less liable to error than those obtained in the usual manner, as long as the dip exceeds 45°; and that, in our latitudes, the accuracy of the new method is nearly three-fold that of the old.

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* *The same number of readings is taken in the two cases, while in the proposed method the process of reversing the poles is dispensed with.*